Hue manifold

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It is generally accepted that hues can be arranged so as to make a circle. The circular representation of hue has been supported by multidimensional scaling, which allows for the representation of a set of colored papers as a configuration in a Euclidean space where the distances between the papers correspond to the perceptual dissimilarities between them. In particular, when papers of various hues are evenly illuminated, they are arranged in a one-dimensional circular configuration. However, under variegated illumination we show that the same papers in fact make a two-dimensional configuration that resembles a torus. © 2010 Optical Society of America

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1. INTRODUCTION

There are two ways to alter the color of an object. One can change either the object’s surface reflectance or the object’s illumination. For instance, one can make a white surface look yellowish either painting it with yellow paint or using an incandescent light source. Will it be the same yellow? Here, we report on experiments that suggest that these are two different yellow hues. In other words, we argue that the hue palette induced by changing the material properties of a surface differs from that induced by changing the surface lighting. This challenges a major dogma of color science: all hues make a one-dimensional manifold topologically equivalent to a circle.

In fact, there is as much experimental evidence against this dogma as in favor of it. On one hand, it has been established that human observers readily distinguish between lighting- and material-induced object color alterations [1–3], although it is hardly surprising, because as known from everyday life, we are unlikely to mistake colored shadows for material stains. If against the odds this happens, we call it a color illusion [4–6]. Such an ability would be impossible if the color palettes induced by material and light are identical. On the other hand, there is a strong belief that both of the hue palettes have the same circular structure. Since Newton’s time it has been taken for granted that the lights of all the possible hues form a “hue circle.” Also, it has been known for centuries that papers of various hues can also be arranged in a hue circle (see for a comprehensive historical review [7]). Furthermore, when observers are asked to place a set of colored papers on a table so that the distances between them are proportional to their subjective proximities, the typical result is a hue circle.

Representing subjective proximities between objects as distances between points in a Euclidian space of minimal dimensionality is the key idea of multidimensional scaling (MDS). Particularly, non-metric MDS is a technique which takes a matrix of dissimilarities between a number of objects and creates a configuration of points, such that if the proximity between a pair of objects a and b is not less than that between c and d, then the distance between the points representing a and b is not less than that representing c and d [8–10]. MDS analysis corroborates the intuitive conjecture concerning the circular arrangement of the hue palette of lights [9,11,12] and colored papers (see [13]).

It follows that when we present various colored papers lit by daylight against a white background their hues will be arranged in a hue circle. Likewise, when we display various patches of colored lights on the same white background their hues will be also arranged in a hue circle. Are these two (i.e., material and lighting) hue circles the same or different? What happens when both the papers and lights are varied? How will the hues be arranged in this case? We address these questions experimentally using MDS. It should be mentioned that, to our knowledge, MDS has never been employed before to ascertain the dimensionality of the hue palette for variegated scenes with multiple illuminants.

As non-metric algorithms of MDS imply that the distances between colors are in the same order as the corresponding dissimilarities, an ideal method of collecting data would be quadruple comparisons—that is, comparing all pairs of pairs of stimulus colors. In other words, a matrix of quadruple comparisons should, in principle, be used as the input to non-metric MDS algorithms. However, as the method of quadruple comparison is rather time-consuming, experimenters resort to more rapid techniques to quantify color dissimilarities, such as rating single pairs [9,11,14–19]. Rating is particularly efficient because it reduces quadruple comparisons to pair comparisons, thereby decreasing the number of evaluations. As the number of pairs to be assessed in our experiment was large, we also chose to use numerical rating to evalu-
ate dissimilarities. A preliminary experiment was carried out to check whether quadruple comparisons and numerical rating both produce the same results.

2. METHODS

A. Stimuli
The experimental setup was analogous to that used previously [19]. The experimental display consisted of six rectangular fields made of white paper with black random-dot design (Fig. 1). Two fields (1 and 2 in Fig. 1) were used to present the stimulus papers—a sample from the Munsell Book of Color [20]. The Munsell notation of the stimulus papers was 5R4/14, 5YR7/12, 5Y8/12, 5G6/10, 10BG5/8, 5PB5/12, and 10P5/12. The distance between the display and the observers was 2 m. The angular size of the whole display was 26° × 29°, with each paper subtending an angle of approximately 4°. The experimental room was semidarkened. The vision was binocular.

In the preliminary experiment the entire experimental display was evenly illuminated by a neutral light (CIE 1931 chromaticity coordinates x = 0.397, y = 0.411, 110 lx). In the first part of the preliminary experiment all the fields except for fields 1 and 2 were left empty. In the second part of the preliminary experiment and in the main experiment fields 3 and 4 were used to present the standard pair of the stimulus papers (see below), with fields 5 and 6 being empty.

In the main experiment, fields 1 and 2 were illuminated independently by one of five lights (yellow, blue, red, green, and purple), giving a total of 15 possible illuminations (red-red, red-yellow, etc.). A digital projector (NEC MT1050) was used to control the illumination. The lights were approximately equiluminant with CIE 1976 u′v′-coordinates (0.382, 0.488), (0.199, 0.530), (0.127, 0.532), (0.183, 0.210), and (0.259, 0.365). The illuminance fell in the range 59–66 lx. Figure 2(a) shows the CIE 1976 u′v′-coordinates of the 35 lights reflected from all seven of the Munsell papers under each of the five illuminations. Fields 5 and 6 empty of Munsell papers were set up to balance the overall illumination in the experimental display. Their illumination was chosen (from the five lights) so as to reduce chromatic adaptation.

The fields in the experimental display were clearly separated from each other with black thin dividers. They were illuminated so that there were no illumination borders between spotlights. When the lights were different, all the observers had clear understanding that the fields are differently illuminated. Specifically, they saw real colored papers lit by real lights of different colors.

B. Procedures of the Preliminary Experiment
In the first part of the preliminary experiment quadruple comparisons were used as the judgment method. The experimenter pointed out at random two different pairs of different papers (one in field 1 and the other in field 2), asking in which pair the papers were more dissimilar in color. Each pair of pairs (210 in total) was evaluated ten times by each observer. The number of times the pair was preferred to all other pairs as most dissimilar was taken as the measure of dissimilarity between papers (for more details, see [21]).

Then, in the second part of the preliminary experiment, the method of rating was used for judgment. Two different papers in different fields were singled out randomly by the experimenter. A pair of yellow (5Y8/12) and blue (10PB5/12) papers was set up as a standard of dissimilarity. It was permanently present in the observer’s view (fields 3 and 4 in Fig. 1). Observers were asked to evaluate the dissimilarity between the papers as compared to the standard pair using a number, taking the standard as 100. Each pair was evaluated five times by each observer. Five normal trichromatic observers took part in the preliminary experiment.

C. Procedures of the Main Experiment
In the main experiment, the method of rating was used as judgment. The experiment was divided into sessions. In each session a fixed pair of chromatic lights was used. A session consisted of 49 trials in which all the 7 × 7 pairs of papers were evaluated. In each trial two papers in different fields were indicated randomly by light-emitting diodes next to each paper. Observers were first asked whether they looked the same or different. All the observers found the papers to appear different. (Admittedly, the same papers lit by the same light looked alike.) Then, they were asked to evaluate the dissimilarity between the papers as compared to the standard, taking as the standard the yellow paper (5Y8/12) under the yellow illumination and the blue paper (10PB5/12) under the blue illumination (fields 3 and 4 in Fig. 1). Each session was repeated six times for each observer.

Out of the five observers who took part in the preliminary experiment, two went on to take part in the main experiment. They completed 15 sessions (i.e., for all possible combinations of five illuminations). We also used a control
group of three more observers, who conducted only ten
sessions (all the combinations of the yellow, blue, red, and
green illuminations). Results for all the observers were
similar. Only the data obtained for the two observers who
carried out all 15 sessions are presented in this report.

3. RESULTS

The dissimilarities obtained in both parts of the prelimi-
nary experiment were averaged separately across ses-
sions and the five observers who took part in the prelimi-
nary experiments. Then the averaged dissimilarity
matrices were analyzed by using a non-metric MDS algo-

rithm. In order to establish how the output configurations
obtained by different methods related to each other, we
conducted a Procrustes analysis [10], which performed
the permissible transformations (dilating, translating, re-
flecting, and rotating) on the rating-based configuration
to bring it as close as possible to the quadruple-
comparison-based configuration. The transformed con-
figurations, along with the output configurations pro-
duced by quadruple comparisons, are shown in Fig. 3. The
Procrustes statistic [10], p. 127] has been evaluated as
0.026, which was small enough to justify the use of the
rating method in the main experiment (the smaller the
statistic value, the better match). While it is not quite cir-
cular, the circular order of the colors therein is the same
as in the Munsell book. A similar somewhat distorted
“hue circle” emerged in many early MDS studies of Mun-
sell papers (e.g., [13,22,23]). The output configuration in
Fig. 3 will be referred to as the material-hue contour.

The dissimilarities collected in the main experiment
were averaged over six repeats (i.e., sessions) and two ob-
servers. A resultant (average) matrix (35×35) of dissimi-
larities was entered into the same MDS algorithm. Figure
4 shows a three-dimensional (3D) non-metric MDS solu-
tion. Stress (i.e., an index showing the relative proportion
of mismatches between the distances and the dissimilar-
ities) was evaluated for two-dimensional (2D) and 3D out-
put space as 0.12 and 0.09, respectively. The smaller
value for the 3D space indicates that the 3D output con-
figuration better renders the dissimilarity structure. A
standard assumption underlying non-metric MDS of color
stimuli is that observers’ dissimilarity judgments are
monotonically related to the subjective distance between
the stimuli [24]. Numerous previous MDS studies on
Munsell papers homogeneously illuminated [13] corrobo-
rate this assumption. Hence, it is safe to believe that
Fig. 4 represents (up to some unknown order-preserving
transformation) the configuration the stimulus papers
make in the subjective color space.

In Fig. 4 the points representing the same paper illu-
ninated by different lights [connected by a line of the
same color as in Fig. 2(b)] make a closed contour which is
filled with a color similar to the color of the paper under
daylight. These contours will be referred to as lighting-
hue contours since they show how a paper’s position in the

![Fig. 2.](image) Chromaticity of the lights reflected from the Munsell papers under various illuminations presented in the CIE
1976 uniform chromaticity diagram. The shape and face color of the markers specify the color of the paper. [Circle—5R4/14, triangle
(up)—5YR7/12, triangle (pointing right)—5Y8/12, square—5G6/10, pentagram—10BG5/8, triangle (pointing down)—5PB5/12,
diamond—10P5/12.] The edge color of the markers specifies the color of the illumination. (a) The Munsell papers lit by the same light are
connected with a line whose color denotes the color of the illumination. (b) The points representing the same Munsell paper under five
different illuminants are connected by a line whose color resembles the color of the paper under daylight.

![Fig. 3.](image) MDS output configurations (material-hue contours) obtained in the preliminary experiment. The
quadruple-comparison-based configuration is represented by the
joined-up colored symbols. The transformed rating-based con-
figuration is marked by crosses. Notation of colored symbols is
the same as in Fig. 2.
dissimilarity space alters when the paper’s illumination varies. The lighting-hue contours are arranged along a closed horizontal contour represented by a gray filling in Fig. 4. This contour is made by the centroids of all the lighting-hue contours (hexagrams). Note that its form is close to that in Fig. 3. Because of this, and also because the shift along the horizontal contour is induced by variations between papers, we believe that the horizontal contour represents the material-hue contour.

As the configurations in Figs. 2(b) and 4 are rather different, the dissimilarities between papers are unlikely to be determined by the chromaticity differences between the reflected lights (i.e., by the distances between the corresponding points in Fig. 2). Indeed, Fig. 5 shows that the dissimilarity judgements (especially for different pairs of lights) are poorly correlated with the corresponding distances in the CIE 1976 uniform chromaticity diagram. For example, the blue paper (5PB5/12) under the purple light and the red paper (5R4/14) under the blue light have nearly equal chromaticity coordinates (Fig. 2). Yet, they were perceived as a great deal dissimilar. More specifically, the dissimilarity between these papers was as much as 70% of the maximal dissimilarity (which was observed between the red paper under the red light and the blue paper under the blue light). Therefore, in a multiple illuminant scene the color of an object cannot be reduced to the color of the light reflected from its surface.

It is important to point out the transversality of the lighting-hue and material-hue contours that can be clearly seen in Fig. 4(b) where a different view of the same configuration as in Fig. 4(a) is presented. The lighting-hue contours in Fig. 4(b) apparently protrude upward and downward from the material-hue contour plane. Therefore, whereas in Fig. 2 the “paper” chromaticity contours [Fig. 2(a)] and the “light” chromaticity contours [Fig. 2(b)] are located in the same plane, the material-hue and lighting-hue contours in Fig. 4 are transversal to each other.

4. DISCUSSION

The present results are in accord with the previous studies where the existence of the material and lighting dimensions has been shown for object colors [18,19,25,26].

![Fig. 4. (Color online) (a), (b) Two views of the output configuration produced by the non-metric MDS algorithm obtained in the main experiment. Each point represents a Munsell paper illuminated by a particular light. Notation is the same as in Fig. 2.](image1)

![Fig. 5. Dissimilarities versus chromaticity differences. In each plate the vertical axis is the dissimilarity between a pair of Munsell papers illuminated separately by one of the five lights: yellow (Y), blue (B), red (R), green (G), and purple (P); the horizontal axis is the chromaticity difference between the reflected lights evaluated in terms of the CIE 1976 uniform chromaticity diagram. Dissimilarities (respectively, chromaticity differences) of pairs were normalized by the dissimilarity (respectively, the chromaticity difference) of the pair (5Y8/12, 5PB5/12) being used as the standard (see Fig. 1), generated under the lighting conditions in question (i.e., Y-B, P-R, etc.) for each observer individually.](image2)
Particularly, Logvinenko and Maloney [18] showed that achromatic Munsell papers illuminated by neutral lights of different intensities produced a 2D MDS output configuration. This configuration comprised regular arc-like layers parallel to each other. The papers illuminated by the same light located in the same layer. Different layers corresponded to different illuminations. It was concluded that the dissimilarity along the layer reflected the perceptual difference in lightness, whereas the dissimilarity across the layers reflected the perceptual difference in surface-brightness. Lightness and surface-brightness were understood as the material and lighting dimensions of achromatic object colors [18].

It should be pointed out that observers in this experiment judged as most dissimilar two pairs: (i) the black paper lit by the brightest light versus the white paper lit by the dimmest light and (ii) the white paper lit by the brightest light versus the black paper lit by the dimmest light. Interestingly, the papers in the first pair reflected the lights of practically equal luminance, whereas the luminance of the lights reflected by the papers in the second pair differed by a factor of 85. It follows that the dissimilarity judgments in this experiment were not based on the brightness of the reflected light. Thus, the surface-brightness is different from the reflected light brightness.

A similar fan-like structure was discovered in the follow-up experiment with yellow, neutral, and blue Munsell papers [19]. It follows that the new dimension (i.e., surface-brightness) that has emerged in these experiments is not something specific to the achromatic domain, but can be interpreted as the achromatic lighting dimension common to the achromatic and chromatic object colors.

In a subsequent experiment we used yellow, neutral, and blue lights to illuminate the same yellow, neutral, and blue Munsell papers [25]. In fact it was a replication of Logvinenko and Maloney’s study [18] except that the achromatic dimension was replaced with the yellow-blue dimension. The seven papers were chosen so that the reflected light chromaticities fell on the straight line (along the yellow-blue direction) in the CIE chromaticity diagram. Moreover, alternation over the chosen three illuminants did not violate the linearity of the paper chromaticity loci. Specifically, the alternation of illumination resulted in only a shift of the papers chromaticity along the same yellow-blue direction. The illuminants were adjusted so that the reflected lights from (i) the yellow paper lit by the blue light, (ii) the gray paper lit by the neutral light, and (iii) the blue paper lit by the yellow light were practically metameric (that is, they have very close CIE chromaticity coordinates and luminance values).

The results were, in a sense, similar to those obtained in the achromatic domain [18]. Specifically, the MDS output configuration consisted of three slightly zigzagging lines shifted parallel each other in the direction transversal to the lines. Equally illuminated papers lay on the same line. Therefore, it was concluded that each line represented the material yellow-blue continuum. The transversal direction was interpreted as representing the lighting yellow-blue continuum. Notably, the dissimilarity between the yellow paper lit by the blue light and the blue paper lit by the yellow light was found to be nearly maximal despite the chromaticity difference between the lights reflected from these two papers being negligible. This indicates that the lighting dimension revealed in this study does not correlate with the chromaticity of the reflected light nor does it with the reflected light luminance [25]. It should be mentioned that only two hues—yellow and blue—were employed in the experiment [25]. Although the existence of lighting hues had been proved, much remained to be investigated about the lighting-hue dimension. What actually varied along the lighting dimension discovered in this experiment was the lighting chroma for the lighting yellow and blue hues. The next clear step was to vary independently the paper and light chromaticities. In [26] seven Munsell papers of different hues (same as in the present study) and the yellow, neutral, and blue illuminations were used. That experiment brought about three material-hue configurations each similar to that in Fig. 3 shifted approximately parallel to each other in the transversal direction. However, as in the previous experiment [25], this direction could also be interpreted as that of lighting chroma.

The rationale (and the novelty) of the present experiment was to use more illuminations of various chromaticities so as to allow a hypothetical lighting-hue dimension, if any, to reveal itself. The fact that the new dimension that emerged in the present experiment is bent into a closed contour proves that it is most likely the lighting-hue dimension. At any rate, it is neither the surface-brightness dimension nor the lighting chroma dimension discovered in the previous experiments [18,19,25,26].

Note that the lighting-hue contours were found to be smaller than the material-hue contours. This is in apparent contrast with Fig. 2 where the light chromaticity contours [Fig. 2(b)] are much larger than the paper chromaticity contours [Fig. 2(a)]. More specifically, the average length of the paper chromaticity contours in Fig. 2(a) has been evaluated as 0.5, whereas the average length of the light chromaticity contours in Fig. 2(b) is 0.85. Therefore, on average, the paper chromaticity contours in Fig. 2(a) are shorter than light chromaticity contours in Fig. 2(b) by a factor of 1.71. On the other hand, in Fig. 4 the average length of the material-hue contours has been evaluated [27] as 71.7 versus the average length of the lighting-hue contours being 44.6. Therefore, on average, the material-hue contours in Fig. 4 are longer than the lighting-hue contours by a factor of 1.61. This observation corroborates our the previous finding that the same difference in the CIE coordinates produces a significantly larger subjective difference in the material, as compared to lighting, color dimensions [18,25,26]. This phenomenon was called the “lighting discounting” [25]. As we can see, the lighting discounting has been observed in the present experiment as well. The ratio of the material versus lighting-hue contour average lengths (i.e., 1.61) over the ratio of the paper versus light chromaticity contour average lengths (i.e., 1/1.71=0.58) is 2.75. It can be taken as a quantitative index of the lighting discounting obtained in the present study.

The existence of lighting color dimensions has relevance to the problem of simultaneous color constancy. Indeed, it follows from our results that contrary to the gen-
eral belief that the color of an object remains constant when its illumination varies [28–31], the object color does vary with illumination. Note, however, that in our experiment this light-induced color variation took place mainly in the dimension transversal (i.e., along the lighting-hue contour) to the dimension along which the color alteration induced by a material (reflectance) change occurs. Because of such transversality, an asymmetric color match is, generally, impossible—a fact pointed out before [18,32,33]. Indeed, an asymmetric color match is underlain by an implicit assumption that, by manipulating reflectance, one can equate the color appearance of two objects under different illuminations. As one can see in Fig. 4, this is impossible because given a lighting-hue shift (within the same lighting-hue contour), any change in reflectance will only bring about an additional transversal shift (along the material-hue contour), thereby increasing dissimilarity. It must be pointed out, however, that although exact asymmetric color match is impossible, numerous experiments on color constancy testify that “approximate” across-illuminant matching can be readily achieved. This is reflected in such definitions of color constancy as “relative stability” (e.g., [34]) or “qualitative preservation” [35] of object color appearance despite illumination variation. However, the problem is what exactly is meant by these. We argued elsewhere that there is every indication that it is the material color dimensions along which the object color remains practically constant when the illumination changes [36].

At first glance, our results corroborate a belief, dating back to the 19th century, that one can see separately and simultaneously the color of an object and the color of the illumination [37,38]. In line with this, MacLeod recently suggested that one needed six numbers for a color stimulus to be specified: the tristimulus values of the incident light and the tri-stimulus values of the spectral reflectance [39]. Although it is true that the object color stimulus can be specified with six numbers [40], this sextuplet cannot be separated into the two independent triplets of the tristimulus values (for lights and objects) because of the so-called metamer mismatching [40]. It has been proved that both triplets constituting these six numbers are determined by a pair object/light, not object separately and light separately [40].

If the material-hue contours are taken to represent the object color, and the lighting-hue contours the illumination color, then all the lighting-hue contours must be of the same form because they represent the colors of the same five lights. However, in reality they are rather different (Fig. 4). The difference between them cannot be reduced to random fluctuations of observers’ responses. Figure 6 presents the pairs of identical papers lit by different lights. The dissimilarity between the identical papers is induced only by the illumination’s chromaticity difference. Therefore, if lighting-hue were the hue of illumination, then the dissimilarity between all the pairs of identical papers under the same illumination condition (i.e., within the same plate) should be approximately equal. However, this is the case only for the yellow-purple illumination condition (the Y-P plate in Fig. 6). The Friedman test (two-way non-parametric analysis of variance) performed on a subset of data generated only by the pairs of identical papers showed a significant effect of paper ($\chi^2 = 94.4, p < 0.001$). Hence, lighting hue is not illumination hue. Both lighting and material hues are attributes of object color.

If lighting and material hues were separable dimensions, and both the lighting and material hues could be represented as a circle, then—ideally—the hue manifold for object color could be a torus (i.e., the boundary surface of a doughnut-like body). Although the configuration in Fig. 4 can be likened to a body with a doughnut-shaped surface, this “doughnut” is quite irregular. This testifies against that lighting and material hues are separable. Still, the set of all the hues makes a 2D manifold that can be considered as a bundle of the lighting-hue contours or as a bundle of the material-hue contours. However, this manifold cannot be factored into two one-dimensional contours. Thus, the hue manifold for object color is essentially 2D and cannot be reduced to the Cartesian product of the two one-dimensional (lighting and material) hue contours.

It follows that any opaque object can have a variety of hues dependent on that object’s illumination. For example, there is the whole continuum of whites. A white surface under daylight renders only one shade of these. Varying the illumination, one can produce white under yellow illumination, white under blue illumination, and the like: these constitute the lighting continuum of whites. All these whites are readily seen as white. Like-

![Fig. 6](Color online) Dissimilarities versus chromaticity differences for the pairs of identical papers. Colors of symbols and shapes are the same as in Fig. 2. Figures show how the dissimilarities, produced only by the difference in illumination, correlate with the corresponding chromaticity differences.
wise, there is a continuum of blues. As pointed out above, we effortlessly discriminate a blue surface under yellow light from a yellow surface under blue light even though they reflect lights of the same chromaticity [25]. This indicates that lighting blue hue and material blue hue are qualitatively different perceptual experiences (see more on this issue in [25]).

To summarize, when illumination is fixed, the set of chromatic hues for object color is one-dimensional and can be geometrically represented as a slightly distorted circle. Yet under varying illumination, the object color hues make a somewhat distorted torus, being essentially 2D.

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REFERENCES AND NOTES

27. When evaluating material (and lighting) hue contour length we used dissimilarity values obtained in the experiment rather than the distances between the points in Fig. 4.